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RESEARCHES ON THE EARTH'S INTERIOR*

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Abstract

An account is given of the derivation of the distributions of density, pressure, gravitational intensity, incompressibility and rigidity in the Earth's interior. Evidence on the solidity of the inner core is presented. Reference is made to the internal structures of the terrestrial planets and to a theory of the origin of the Moon. Attention is given to some questions of scientific inference.

This is a great occasion for me. As some of you will know, I once had the privilege of being a Victorian resident for several years. Those years were among the most pleasant of my academic life, and, Sydney's rugged charms notwithstanding, I still look back to my time in Victoria with fond memories. The award of this handsome Research Medal of the Royal Society of Victoria therefore means even more to me than the very great honour it confers.

The principal field of my research has been in what is nowadays called theoretical solid-Earth geophysics. The adjective 'solid-Earth' means the Earth from the surface downward and is used to distinguish that part of the Earth from the atmosphere and regions above. The fact that some of the 'solid Earth' happens to be fluid has not worried the inventors of the term. For the most part, my work has been concerned with the mechanical and elastic properties of the Earth's deeper interior, and that will be my central topic tonight.

I shall also be saying a little about my work on the Moon and some of the other planets. Strange as it may seem, this also comes under the heading of solid-Earth geophysics, the reason being that the internal structures of the Earth and other planets are likely to be intimately related. Even before artificial satellites became subjects of serious conversation, geophysicists had been interested in planetary interiors. With satellites now starting to gather direct observations on planets, the interest has lately been intensified.

My approach to my subject is as an applied mathematician, and I shall try, here and there, to give some glimpses of the type of thinking an applied mathematician brings to bear in his research. That does not mean that I shall be attempting to use any serious mathematics this evening. While formal mathematics is of course normally involved in applied mathematical work, applied mathematics is far from being the same subject as pure mathematics. An applied mathematician has, first and foremost, to be context-minded, and pays attention to questions of inductive as well as deductive inference. Theoretical geophysics is an unusually good field to exhibit the applied mathematical approach, and I hope you will not mind if, in addition to relating some specific results of my work, I attempt to show something of the underlying philosophy of the methods used.

I shall first talk about the Earth's interior generally, as a preliminary to showing where my own contributions fit.

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Mean Density and Moment of Inertia

It first became possible to make inferences about the interior of the Earth when the shape, size and mass became determined to reasonable precision. By 1798, Cavendish had obtained a value near $5\frac{1}{2}$ g/cm^3 for the Earth's mean density. This showed that the material of the Earth's deeper interior must be at least twice as dense as the average surface rock.

The inferences were carried further during last century when the Earth's moment of inertia was determined. The moment of inertia of a body is connected with the internal distribution of mass. For a sphere of constant density, the moment of inertia I about a diameter is $0{\cdot}4\ MR^2$, where M and R are the mass and radius. For a body like the Earth, where the density must (essentially) increase with depth, knowledge of the amount by which the coefficient is less than 0·4 contributes to knowledge of the degree of central concentration of mass in the body.

A variety of evidence is used in determining the value of the coefficient for the Earth. As the Earth rotates in the gravitational fields of the Sun and Moon, the Earth's equatorial bulge causes the axis of rotation to precess. The axis describes a cone relative to the distant stars, and the effect reveals itself in an observed motion of certain points (the equinoctial points) around the celestial equator, at a rate just over 50 seconds of arc per year. The phenomenon is called the precession of the equinoxes, and observations of it contribute to the determination of I. Further information on I comes from the theory of the figure of the Earth, in conjunction with knowledge of the surface gravity and speed of axial rotation.

In this way, it was found last century that $I = zMR^2$, where z lies between 0·33 and 0·34. This result confirmed the presence of a substantial increase of density with depth in the Earth, and indicated the order of magnitude of the central concentration. Within the last few years, observations of the orbits of artificial satellites have supplied more precise information on I, and the coefficient z is now estimated to lie between 0·330 and 0·331.

Evidence from Seismology

Any picture of the Earth's internal structure must agree with the known values of the mass, radius and moment of inertia. But there are many possible distributions of mass compatible with these values. To narrow the possibilities requires evidence from fresh sources. The outstanding source of new evidence has been seismology, the study of earthquakes, through which it has become possible to obtain numerical values of certain mechanical properties at specific points throughout the Earth. Through seismology, it has become possible to chart out the structure of the Earth in considerable detail.

With scientific detachment, I shall look upon earthquakes as useful generators of seismic waves, passing over the fact that some of the most 'useful' earthquakes have been among the most disastrous to mankind. Every year there occur many earthquakes large enough to send detectable waves through all parts of the Earth's interior. The waves, on emerging at the surface, are recorded by seismographs in practically all countries of the world. The records, or seismograms, provide the basic observational material from which the seismologist works. In a sense, seismology can be looked upon as a branch of communication theory. An earthquake conveys to receivers at the surface information on the nether regions its waves have traversed. It writes its information on the world's seismograms, and a task of the seismologist is to decode the writing.

The first requirement in the decoding process is a reliable theory of seismic wave transmission. The waves are of the nature of mechanical vibrations. As they

pass through the Earth, they involve to-and-fro motion of particles, and thus deformational motion inside the Earth. So what is required is a suitable theory on the deformation of the internal material of the Earth—a theory on the inter-relation of stress and strain.

Elasticity Theory

A typical question of scientific method now enters. Experiments have shown that the relatively simple mathematical theory of perfect elasticity, a generalization of Hooke's law, gives a useful description of the observed elastic behaviour of many solids and fluids subjected to stress under ordinary laboratory conditions.

A characteristic of perfect elasticity in its simplest form is that, for a given material under given thermodynamical conditions, the stress-strain relations are entirely determined by knowledge of just two coefficients, which may be suitably taken as the 'incompressibility' or 'bulk-modulus', and the 'rigidity'. The incompressibility specifies the capacity of the material to resist a symmetrical pressure; rigidity, the capacity to resist stresses tending to change the shape. Perfectly elastic materials are classified into solids and liquids according to the magnitude of the rigidity. In ordinary conditions, a solid has considerable incompressibility and rigidity (of the order of 10^{11} dyn/cm^2 or more) while a fluid has negligible rigidity. Thus, for example, a fluid can readily change its shape to fit a containing vessel.

The equations of perfect elasticity, like all equations and so-called laws in natural philosophy, are mathematical models which could apply with total precision only to ideal materials. The utility of any mathematical model is gauged by its degree of simplicity and its compatibility with the relevant observations. The question arises as to the suitability of the perfect elasticity model for the unknown materials of the deep interior of the Earth. Because of its comparative simplicity and its degree of success with laboratory materials, it is appropriate to take this model as a trial for the Earth, to be modified of course should subsequent testing against observations reveal discrepancies. It transpires that the perfect elasticity model is a remarkably good one where seismic observations are concerned, and, in particular, is quite adequate for the principal conclusions I shall be drawing tonight.

As a result of this adequacy, it is permissible to speak of solid and fluid regions of the Earth's interior. This needs stressing because of unnecessary arguments that have sometimes arisen between geologists and applied mathematicians on the point. Geologists and others have, in particular, questioned the use of terms such as solid and fluid where pressures are as high as they are in the Earth's deep interior. But the scientific approach to the description of properties in the Earth is really no different from that for materials in the laboratory. In both cases, a material is called solid or fluid according to the values, large or small, of coefficients which appear in trial equations used to describe the elasticity. The only difference in practice is the irrelevant one of the technique used in finding the values of the coefficients. For materials in the laboratory, the technique is to match the equations against fairly direct laboratory measurements of stress and strain, and for the Earth's interior, to match them against seismic and related observations.

One caution needs, however, to be stated. The periods associated with seismic wave motion do not exceed the order of an hour, and it cannot be asserted from the seismic observations that the perfect elasticity model is adequate in contexts where much longer periods are involved, for example, periods on the geological time scale. More than two coefficients may then be needed in expressing the stress-strain relations, and a simple classification into solid and fluid may not then be

justified. Thus it needs to be appreciated that my use of the terms solid and fluid is part of a description of the essential response of the Earth to comparatively short-lived stresses. I have gone into a little detail on this question, since a section of my research is concerned with the solidity or fluidity of the Earth's deep interior —in the sense just defined.

Seismic Travel-Time Tables

Next for a little detail on the transmission of seismic waves. The perfect elasticity model being accepted as suitable for the context, it can be deduced that the Earth can transmit two types of waves through its interior. These are the 'primary' or *P* waves, and the 'secondary' or *S* waves. The speeds of travel, α and β, of the *P* and *S* waves at any point *Q* are given by

$$\alpha^2 = (k + 4\mu/3)/\rho \,, \qquad \beta^2 = \mu/\rho \tag{1}$$

where ρ, k and μ denote the density, incompressibility and rigidity of the material at *Q*.

The *P* waves are like sound waves and cause the particles of the medium to vibrate in the line of wave advance. The *S* waves, which travel more slowly, cause the particles to vibrate sideways. The *P* waves travel through solids about $1\frac{1}{2}$ times as fast as *S* waves, while *S* waves, which depend crucially on the rigidity, do not travel through fluids. Thus detection of *P* and *S* waves in an internal region of the Earth is evidence of solidity. Failure to detect *S* waves suggests fluidity. Near the Earth's surface, *P* waves travel at about 5 km/sec and *S* waves at about 3 km/sec. The speed of *P* waves reaches its maximum of $13\frac{1}{2}$ km/sec at a depth of 2,900 km below the surface.

In 1897, *P* and *S* waves were identified on seismograms by the Englishman R. D. Oldham. This supplied the first check on the suitability of the perfect elasticity theory for seismic wave transmission in the deep interior of the Earth.

Soon after Oldham's discovery, efforts were put into obtaining tables for the travel times of seismic waves between an earthquake focus and points of the Earth's surface where the waves are recorded. The energy in the waves can, to good approximation, be treated as travelling out along rays analogous to rays of light. This is a consequence of certain common features in the mathematics underlying the transmission of optical and seismic waves. In general, the ray theory is very accurate in localities not too close to the focus.

Seismic rays are reflected or refracted in accordance with a law that is a generalization of Snell's law. When a *P* or an *S* ray meets a surface of discontinuity in the Earth, it may be reflected and refracted into both *P* and *S* rays. This multiplicity of derived rays means that seismograms usually show many 'phases', each phase corresponding to the arrival of a group of waves by one of many types of path. Between surfaces of discontinuity the rays are generally curved. This is because the seismic velocities α and β change continuously with depth as ρ, k and μ change.

The evolution of travel-time tables involves sorting out the many phases on the tangled seismic records, with the help of the basic elasticity theory, wave theory and statistical theory. The task is complicated, among other things, by ignorance of the origin time and source location of every natural earthquake used for the purpose.

At the time when I entered seismology, in 1931, much work had previously been done on travel-time tables, but it was suspected that the tables still contained large errors. The grounds for suspicion were fairly strong. With some particular

earthquakes, for example, calculations based on the existing travel-time tables in fact gave what were euphemistically called 'high foci'—some of the computed foci were 100 to 200 km above the Earth's surface. It was my good fortune to meet Sir Harold Jeffreys of Cambridge at a time when he had decided that a comprehensive revision of the tables was necessary, and between 1931 and 1939 he and I spent a good part of our research energy working jointly on the problem. By 1939, we had managed to reduce the table errors from the order of a minute, in travel times up to 20 minutes and more, to the order of a small number of seconds. We were not the only workers on the problem. Gutenberg and Richter in California also spent some years seeking to improve the travel times, and their results agreed with ours within the claimed accuracy. Since 1940, the tables of Jeffreys and myself, called the J.-B. tables, have been used in preparing the International Seismological Summary.

Use of Nuclear Explosions

Finer accuracy in travel times is now being obtained through the use of nuclear explosions, which are also generators of seismic waves. Moreover, in spite of being smaller than large natural earthquakes, they have the cardinal advantage of providing controlled experiments in which the location of the focus and the time of origin can be precisely known.

In 1957, the late T. N. Burke-Gaffney of Riverview Observatory, Sydney, and myself brought a degree of notoriety upon ourselves by estimating, for four American hydrogen bomb explosions, origin times which, when the source data were later released, proved to be correct to 0·0, 0·4, 0·7, and 0·1 second, respectively. From that time, the United States has released source details on most American nuclear explosions, with much advantage to the science of seismology.

Comprehensive corrections to the J.-B. tables have, however, not yet been evolved, and the tables continue to be widely used. This is partly because of severe restrictions on the locations of nuclear explosions to date.

The travel-time tables provide the most precisely determined evidence available on the structure of the Earth's interior. An early result was the fairly high degree of independence of the travel times on geographical region, showing that, to a strong first approximation, the matter in the Earth is symmetrically distributed about the centre. The biggest deviation is that due to the Earth's oblateness. Jeffreys and I were able to show, incidentally, that from seismic data alone we could estimate the Earth's ellipticity of figure within an error of order one-sixth. Other deviations affecting the travel times are connected with the Earth's major geographical features and may possibly be significant down to a few hundred km below the surface.

With spherical symmetry established as a serviceable first approximation, it becomes possible by mathematical processes to infer the values of the seismic velocities α and β in terms of the distance r from the Earth's centre. The Earth can then be divided into a number of internal regions according to depth. Boundaries are taken where α and β or their gradients show sudden changes with respect to r.

Further, when the values of α and β are known, the equation (1) shows that the quantities k/ρ and μ/ρ are known. In this way, seismology has provided fairly direct information on these quantities throughout much of the Earth's interior.

Some of the main boundaries below the Earth's surface had been located long before the work on travel times had reached its present degree of precision.

The Earth's Crust, Mantle and Core

In 1909, A. Mohorovičić, a Balkan seismologist, located a boundary some tens of kilometres below the Earth's surface in his region. Others later showed this boundary, now called the Mohorovičić discontinuity, to be world-wide. It is characterized by a marked jump in the *P* and *S* velocities about 35 km below the surface in continental shield areas, is somewhat deeper under some mountain ranges, and shallower under the main ocean floors. The name Mohole has been given to a project to bore a hole down to and through the Mohorovičić discontinuity. In 1957, I happened to be chairman of a meeting in Toronto at which the proposal was first sponsored by the International Association of Seismology. The idea is to obtain samples of materials all the way down to the discontinuity and of the material immediately below, and so provide a useful check on a section of seismological and other inferences.

The region between the Earth's surface and the Mohorovičić discontinuity is nowadays conventionally called 'crust'.

Another early seismological result was Gutenberg's location in 1914 of a major discontinuity at a depth near 2,900 km below the surface. This boundary separates what has come to be called the Earth's 'mantle', on the upper side, from the 'central core' below.

Both *P* and *S* waves are recorded throughout the mantle, showing that the mantle is solid in the sense that I have defined. Further, the rigidity steadily increases with depth throughout nearly all the mantle. A century ago, Kelvin had shown that, contrary to the prevailing view that the Earth is mostly molten below the crust, the average rigidity of the whole Earth exceeds that of ordinary steel. Kelvin's inference was based on astronomical measurements of movements of the Earth's poles and measurements of the tidal deformation of the solid Earth. Kelvin's general conclusion is now amply confirmed with the addition of evidence from seismology. My own calculations give steadily increasing values for the rigidity throughout most of the mantle and a value at the bottom of the mantle between three and four times that of ordinary steel.

This last result, taken in conjunction with modern evidence of the type used by Kelvin, in fact leaves little room for any rigidity in the central core. The Japanese geophysicist Takeuchi in 1950 and the Russian Molodenski in 1955, as a result of very arduous calculations, showed from the overall evidence that the average rigidity below the mantle can at most be a small fraction of the value in the mantle. Their work confirmed, what had been long suspected, that most of the central core is in a fluid or molten state. The result is in line with the failure of seismologists ever to detect *S* seismic waves in the core.

The Earth's Inner Core

An important discovery was made in 1936 by the Danish woman seismologist, Inge Lehmann. Using European records of two New Zealand earthquakes and some other evidence, she inferred the existence, deep down in the Earth's central core, of an inner core characterized by a marked jump in the seismic *P* velocity. The inner core is comparatively small, with radius about 1,200 km. For want of a better name, the part of the old 'central core' outside the inner core has been called the 'outer core'. The outer core is almost certainly molten, but I shall later give reasons which have led me to think that the inner core is probably solid. (The calculations on the small average rigidity of the central core are not fine enough to enable a conclusion to be drawn about the fluidity or rigidity of the inner core, which occupies only about 1 per cent of the whole core volume.)

A broad picture of the interior of the Earth as revealed by seismology is thus: a thin crust at the surface; a solid mantle extending downward for 2,900 km; a fluid outer core, some 2,200 km thick; and then the inner core, probably solid, occupying the remaining 1,200 km to the centre.

The Earth's Density Distribution

I shall now say something about my work on the Earth's density and related properties. This has been my longest-sustained interest, and started in 1935 at the time when I was working with Jeffreys on the travel-time tables. We had then reached the point where it became necessary to take note of the effect of the Earth's oblateness on the seismic travel times, and I undertook to estimate the necessary allowances.

It turned out that this was not merely a matter of allowing for extra lengths of path where rays extend through the Earth's equatorial bulge, or for reduced lengths due to the flattening of the Earth in high latitudes. The paths are also affected along their entire length by the ellipticities of internal surfaces of constant density. Before the effects could be calculated it was necessary to know these internal ellipticities to adequate precision. This in turn depended on knowledge of the internal distribution of density in the Earth, and I soon found that the existing density results were far from being reliable enough for my purpose. Thus in 1935 I stopped my other work for a time and set about trying to obtain reliable details on the Earth's density.

There was available a differential equation for the density gradient at points of the Earth's interior, first used, in 1923, by the Americans Williamson and Adams. This equation utilizes the fact that knowledge of the P and S seismic velocities at any depth in the Earth gives knowledge of k/ρ at that depth. On the perfect elasticity theory, k/ρ gives the ratio of the pressure gradient to the density gradient for a material of uniform composition. In this way it is possible to make some progress towards evaluating density gradients in the Earth.

Prior to 1936, the Williamson-Adams equation had been used by a number of authors in seeking to work out the variation of density in parts of the Earth. But the equation involves a number of assumptions outside the seismic evidence, and widely different values of the density distribution had been obtained. It was my good fortune to stumble across a method which appeared to yield precisely determined values for the first time.

I assumed a value near $3{\cdot}3$ g/cm^3 for the density just below the Earth's crust and applied the Williamson-Adams equation, in conjunction with the then available seismic data, to obtain a trial density distribution for the mantle. I then applied a test which had not been previously used. This was to work out the consequent value of the moment of inertia of the core. On the basis of my trial density solution for the mantle, I computed the moment of inertia of the mantle and subtracted this from the known moment of inertia of the whole Earth. The result gave $0{\cdot}57\ ma^2$, where m and a are the mass and the radius of the core. Now the coefficient $0{\cdot}57$, being substantially in excess of $0{\cdot}40$, would require the core to be much denser near the outside than near the centre, a conclusion that has to be rejected on grounds of the Earth's stability.

It followed that the procedure in obtaining the trial density distribution for the mantle was somewhere seriously in error. After exhausting the possibilities I was able to show that the weak point was that, over a sizable range of depth, the Williamson-Adams equation is unreliable in respect to the assumption of uniform composition. I found that I had provided evidence that there exists inside the

mantle a substantial variation of chemical composition, or, alternatively, substantial phase changes.

Further, by making the necessary modifications to my approach, and bringing the moment of inertia test to bear, I was able to obtain what seemed, after all tests had been made, fairly narrowly determined values of the density ρ down to a depth of nearly 5,000 km below the surface, that is, down to near the inner core boundary. The details of the calculation are fairly complicated, and had to be revised when the *P* and *S* velocities came to be better determined from the J.-B. tables. I shall here give only the broad results.

According to these, the density varies from 3·3 just below the crust to 5½ g/cm³ at the bottom of the mantle. The density then jumps to 9½ at the top of the outer core, and reaches 11½ to 12 g/cm³ near the inner core boundary. The results appeared to be accurate within 2-3 per cent throughout the mantle and 5 per cent in the outer core, and this estimate of the uncertainties still stands.

I was also able to show that the density at the Earth's centre must be at least 12·3 g/cm³, but was not able to make a positive estimate of the density in the inner core until much later. In 1942, I worked out several Earth models taking different arbitrary values for the central density. Two of these models continue to be of interest. In one of them, the central density has the minimum value of 12·3 g/cm³; in another, it is 17·3 g/cm³. Because the inner core is so small, there is not much difference between the models outside the inner core.

Pressure, Gravity, Incompressibility, Rigidity

The work on density carried with it a crop of results on other properties, including values for the pressure, gravitational intensity, incompressibility and rigidity, throughout most of the Earth.

I was able to show that the pressure reaches about 1⅓ million atmospheres at the bottom of the mantle, and between 3½ and 4 million atmospheres at the Earth's centre. I found the value of *g* (gravitational attraction) to stay within 1 per cent of 990 cm/sec² down to a depth of 2,400 km, rising to a maximum of about 1,050 cm/sec² at the mantle-core boundary, and then dropping steadily to reach zero at the Earth's centre.

The rigidity values rose fairly steadily with depth in the mantle, reaching about 3×10^{12} dyn/cm² at the bottom, then dropping below measurable limits inside the outer core.

The incompressibility values in general rose steadily throughout the Earth, to 14×10^{12} dyn/cm² or more at the centre.

Confirmation from Chilean Earthquake

In 1960, there was unexpected confirmation of the essential numerical results in this set of calculations. In May of that year, there occurred in Chile a very great earthquake which excited fundamental vibrations of the whole Earth, sufficiently large to be clearly recorded in several parts of the world. There was a spectrum of fundamental and overtone periods, ranging from nearly an hour downwards, and providing invaluable new observational material on the structure of the Earth's interior.

It is possible, by heavy mathematical labour, to calculate the periods of these vibrations for any Earth model, given the distributions of density, incompressibility and rigidity. Such calculations had been carried out for a number of Earth models, including mine, by C. L. Pekeris of Israel. The results showed that the new

observations could be fitted only by models which agreed with mine within the stated uncertainties.

Efforts have since been made to use the Chilean observations to add precision to results on the Earth's density. Some small modifications have been suggested inside the Earth's mantle, but the new method has not yet gone very far beyond providing valuable confirmation of the earlier results.

Solidity of Inner Core

I return now to my calculations on the incompressibility k. In the region consisting of the lower mantle and outer core together. I found that k varies much more smoothly than either the density or rigidity, both of which have large changes at the mantle-core boundary. When I looked into physical aspects of the behaviour of k, I found reason to propose, in 1946, as a trial hypothesis, that k varies smoothly with the pressure p throughout the whole Earth from the middle of the mantle to the centre of the core.

It was this development which led me to infer that the Earth's inner core is probably solid. Stripped of complications, the argument is essentially as follows. Miss Lehmann had shown that α jumps in value from the outer to the inner core. By (1),

$$\rho\alpha^2 = k + 4\mu/3. \tag{2}$$

Almost certainly, ρ does not decrease significantly with depth. In view of the evidence that k varies smoothly, I then interpreted the jump in α as due to a jump in μ rather than k. Since μ is effectively zero in the outer core, I thus inferred that there is probably a change from the molten to the solid state at the inner core boundary.

During the following twenty years, I have had to adapt my theory on compressibility to meet later evidence from experimental and theoretical physics. But the key result still remains that it is very improbable that k could jump sufficiently at the inner core boundary to account for the seismic observations. Although direct evidence has not yet been obtained, it thus remains strongly probable that the inner core is solid. In the last few years I have obtained new evidence which I shall mention shortly.

The Earth's Central Density

It was through the theory on compressibility that I was also able, in 1950, to make a tentative estimate of 18 g/cm³ for the Earth's central density ρ_0. This first estimate depended, however, on values of the seismic velocity gradients in the core, some of which were very uncertain, especially near the inner core boundary, which has proved to be far from a simple boundary. My first estimate of ρ_0 rested on a P velocity distribution of Jeffreys which gave a transition region, 150 km thick, between outer and inner core, characterized by a high negative gradient for α.

The estimate of ρ_0 has had to be substantially modified during the past few years. In our work on hydrogen bomb explosions, Burke-Gaffney and I came across unmistakable evidence of early P wave arrivals at angular distances between 130° and 140° from the explosion source. Certain arrivals were up to 13 seconds earlier than in the J.-B. tables. Around the same time, Gutenberg gave evidence of similar early readings from natural earthquakes.

Dr B. A. Bolt, then a member of my Sydney department, showed in 1961 that all existing seismic observations could be accounted for by including an additional branch in the P travel-time curve for waves in the core. The new branch required the core to contain an additional layer, making four in all—outer core, two tran-

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sitional layers, and then the inner core proper. The last word has not yet been said, but seismologists are agreed that Bolt's new velocity distribution is much superior to the old—so much so that he became speedily translated from Sydney to a major chair in the United States.

In relation to my density problem, an important feature of Bolt's work was that it removed the evidence on negative P velocity gradients in the core. This result led me in 1962 to reduce my estimate of ρ_0 from 18 to 15 g/cm^3.

In the meantime, two further developments have taken place which have led me to carry my density revision further in the lower core. The first was the assemblage, by Francis Birch of Harvard, of a body of evidence which points to a value not exceeding 13 g/cm^3 for the Earth's central density. The evidence includes shock-wave experiments involving short-lived pressures reaching the order of the pressures in the Earth's core. There are some controversial questions on the interpretation of the experiments, but Birch's estimate of 13 g/cm^3 for ρ_0 has to be taken seriously.

The second development was some theoretical work of my own, seeking to extend the Williamson-Adams method to provide a serviceable equation on density gradients in regions of the Earth's deep interior where one cannot assume chemical homogeneity. In 1964, I applied this theory, and came to the conclusion that Birch's figure of 13 g/cm^3 for ρ_0 could be met in only one way.

In making my previous estimates of ρ_0, I had assumed that the S velocity β in the inner core would have similar gradients to the P velocity. But there is no available test of this assumption. Various difficulties have so far made it impossible to detect and measure S waves in the inner core.

I found that I could accommodate a central density of 13 g/cm^3 only by departing from my previous assumption and having a negative gradient for β in the lower core. And a negative S velocity gradient demands a negative rigidity gradient if the density is not to go well beyond 13 g/cm^3 at the centre.

Thus if Birch's work proves to be substantiated, I think I have shown that, after an initial change from the molten to the solid state somewhere well down in the core, there is a trend back towards fluidity as the centre is approached. Moreover, additional evidence is provided on the solidity of the inner core. For there cannot be a negative rigidity gradient unless there is rigidity, i.e. solidity. More specifically, the calculations show that an entirely fluid core would, on the current seismic data, require the central density to be at least $14\frac{1}{2}$ g/cm^3. Part of my time during leave in 1964 was spent working out a number of models for the Earth's lower core exhibiting the range of possibilities.

Planetary Interiors

I shall now touch, perforce briefly, on one or two aspects of my interest in planetary interiors. This interest goes back to 1937 when I used my first density-pressure results for the Earth to see what light might be thrown on the comparative compositions of the Earth and the terrestrial planets. In that year, Jeffreys and I independently showed that if the mantles of the Earth, Venus and Mars are similarly composed, and also the cores, but the mantles chemically distinct from the cores, then the mantle-core mass ratios would have to be very different for the three planets. It therefore seemed that the overall compositions of the planets must be significantly different.

This conclusion was favoured until, over the period 1948-50, W. H. Ramsey of England and I tried a new idea. We independently showed that if the change from mantle to core in the Earth were principally due to pressure rather than

change of composition—if the change were essentially a phase transformation—then one could, to good approximation, fit the available astronomical data and the pressure-density relation for the Earth, and still have the same overall composition for all three planets.

There are, however, some obstacles to this idea. There are complications with the planet Mercury, and geochemists do not favour the phase transformation idea because of the large jump in density at the Earth's mantle-core boundary. Nevertheless, it has not yet proved possible to obtain a decisive test. For a period of years, the theory received little attention, but it has lately come into the news again, notably at the hands of R. A. Lyttleton who has used it in a new theory of the Earth's mountain building.

An interesting aspect of the theory is that it entails a much smaller fluid outer core in Venus than in the Earth, and no fluid zone in Mars. If one accepts the current view that the main seat of the Earth's magnetism is in the fluid outer core, it would then be expected that Venus has a much weaker field than the Earth, and Mars an extremely small field, if any. So far as artificial satellite observations have gone, these results seem to be substantiated.

What is really wanted now is seismograph recordings on the planets. If we could, for example, thereby locate and estimate the thickness of a Venusian fluid outer core, we should be well on the way to checking important inferences on the interior of the Earth. Thus it is that the interiors of planets have become a recognized part of solid-Earth geophysics.

Origin of the Moon

Another part of my extra-terrestrial work relates to the origin of the Moon. With the help of a student A. N. Datta, I showed that in certain circumstances a mass equal to that of the Moon could have been expelled from a primitive Earth-Moon body with explosive violence. This was a consequence of calculations on the gravitational energy of a planet containing a small core in the form of a phase transformation of the material outside.

It would be going too far to claim that these calculations provide more than one of a number of possibilities on the origin of the Moon. At the time they were made, there were heavy arguments on other grounds against the idea that the Moon and Earth were once a single body. But the calculations remain formally valid, and latterly there has been some renewed interest in the idea of a primitive Earth-Moon body.

The Role of Mathematical Models

I should like, finally, to say that I have found research in geophysics to provide a great education in the philosophy of science. Physicists and some other scientists may sometimes think that they obtain factual results by direct observation in the laboratory. Where the deep interior of the Earth is concerned, there cannot be even the pretence of direct observation. This has led to two rather common attitudes, both unscientific. One is a tendency to build geophysical theories on flimsy evidence and, with fashion lending support, to attach quite unwarranted weight to highly doubtful inferences. The other attitude is a tendency on the part of some writers to dismiss results on the Earth's interior as 'inductive'—as if there existed any branch of natural sciences in which the inferences are not all inductive.

The method I have tried to bring to bear is inference through mathematical models. As I see it, all discussion in natural science, whether in geophysics or any other science, is in terms of mathematical models, using the word 'mathematical' in

a very broad sense. I would regard the central aim in science to be the erecting of mathematical models which will describe fields of observational data as accurately and tersely as possible. Formal mathematics does not necessarily have to be used, particularly in the early stages. There are parts of geology, for example, where it would be foolish to apply sophisticated mathematics. Also, mathematics alone must never be allowed to dictate. But the spirit of applied mathematics should, in my view, be present in all scientific discussions.

No mathematical model, whether given the august name of 'law of nature' or not, should ever be confused with reality. The task, as I see it, is to sort the models out into some order of reliability in relation to the evidence available at the time.

In geophysics, it is perhaps fair to say that, due to paucity of crucial evidence, there is more than the usual number of flimsily established models. On the other hand, the properties of some other geophysical models appear to be as reliably established as many ordinary laboratory results. Thus inferences in geophysics appear to me to be of the same general character as in all science. I hope that in this address I have succeeded in giving some indication of the weights I think should be attached to the various parts of my work.

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